Two-Loop $\mathcal{O}(\alpha_s G_F m_t^2)$ Corrections to Higgs Production at LEP

Bernd A. Kniehl

Max-Planck-Institut für Physik, Werner-Heisenberg-Institut
Föhringer Ring 6, 80805 Munich, Germany

Michael Spira

II. Institut für Theoretische Physik,* Universität Hamburg
Luruper Chaussee 149, 22761 Hamburg, Germany

Abstract

We evaluate the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to the ZZH coupling in the Standard Model by means of a low-energy theorem, assuming that the top quark is much heavier than the Higgs boson. We then construct a heavy-top-quark effective Lagrangian for the ZZH interaction that accommodates the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections and derive from it the corresponding corrections to the $H \to ZZ$ decay as well as those to Higgs-boson production at LEP1, via $Z \to f\bar{f}H$, and at LEP2, via $e^+ e^- \to ZH$. In all cases, the leading $\mathcal{O}(G_F m_t^2)$ terms are considerably screened by their QCD corrections, if the on-shell renormalization scheme with G_F as a basic parameter is employed.

1   Introduction

The Higgs boson is the missing link in the Standard Model (SM). The discovery of this particle and the study of its characteristics are among the prime goals of present and future high-energy colliding-beam experiments. At LEP1 and SLC, the Higgs boson is currently being searched for in the decay products of the $Z \to f\bar{f}H$ channel [1]. At the present time, the failure of this search allows one to rule out the mass range $M_H \le 63.9$ GeV at the 95% confidence level [2]. At LEP2, the quest for the Higgs boson will be continued using the Higgs-strahlung mechanism, $e^+ e^- \to ZH$ [3, 4].

Once a novel scalar particle is discovered, it will be crucial to decide if it is the very Higgs boson of the SM or if it lives in some more extended Higgs sector. To that end, precise knowledge of the SM prediction will be mandatory, i.e., quantum corrections must be taken into account. The current knowledge of radiative corrections to the production and decay processes of the SM Higgs boson has recently been reviewed [5]. Heavy-fermion effects on $\Gamma\left(Z \to f\bar{f}H\right)$ and $\sigma(e^+e^- \to ZH)$, at one loop, have been analyzed in Ref. [6]. The full one-loop electroweak correction to $\Gamma\left(Z \to f\bar{f}H\right)$ has been presented in Ref. [7]. The one-loop QED [8, 9] and weak [9, 10] corrections to $\sigma(e^+e^- \to ZH)$ are also available. The theoretical predictions for $\sigma(e^+e^- \to ZH)$ at LEP2 energies have recently been collected and updated [11].

In view of experimental evidence for a heavy top quark, with $m_t = (174\pm16)$ GeV [12], the m_t-dependent corrections are particularly important. In the case of $\Gamma\left(Z \to f\bar{f}H\right)$ and $\sigma(e^+e^- \to ZH)$, they may be accommodated by multiplying the respective Born formulae with $(1 + \Delta_{f\bar{f}ZH})$, where [6, 7, 9]

$$\Delta_{f\bar{f}ZH} = N_c x_t \left(-\frac{2}{3} - \frac{8c_w^2 Q_f v_f}{v_f^2 + a_f^2}\right),\tag{1}$$

which is negative for all flavours. Here, $N_c = 3$, $x_t = \left(G_F m_t^2/8\pi^2\sqrt{2}\right)$, $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$, $v_f = 2I_f - 4s_w^2 Q_f$, $a_f = 2I_f$, Q_f is the electric charge of f in units of the positron charge, I_f is the third component of weak isospin of the left-handed component of f, and it is understood that the Born results are expressed in terms of the Fermi constant, G_F. In the case of $\sigma(e^+e^- \to ZH)$, $f = e$ in Eq. (1).

The goal of this paper is to derive the QCD correction to Eq. (1). This will be achieved by means of an appropriate low-energy theorem [3, 13, 14]. Generally speaking, this theorem relates the amplitudes of two processes which differ by the insertion of an external Higgs-boson line carrying zero momentum. It provides a convenient tool for estimating the properties of a Higgs boson which is light compared to the loop particles. In the literature, a similar theorem has been applied to derive low-M_H effective Lagrangians for the $\gamma\gamma H$ and ggH interactions at one [13] and two loops [14]. In a previous paper [15], we have employed this theorem to find the non-universal $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to $\Gamma\left(H \to b\bar{b}\right)$.

For the reader's convenience, we shall review the gist of the matter. This low-energy theorem may be derived by observing that the interactions of the Higgs boson with the massive particles in the SM emerge from their mass terms by substituting $m_i \to m_i(1 + H/v)$, where m_i is the mass of the respective particle, H is the Higgs field, and v is the Higgs vacuum expectation value. On the other hand, a Higgs boson with zero momentum is represented by a constant field, since $i\partial_\mu H = [P_\mu, H] = 0$, where P_μ is the four-momentum operator. This immediately implies that a zero-momentum Higgs boson may be attached to an amplitude, $\mathcal{M}(A \to B)$, by carrying out the operation

$$\lim_{p_H \to 0} \mathcal{M}(A \to B + H) = \frac{1}{v}\sum_i \frac{m_i\partial}{\partial m_i}\mathcal{M}(A \to B),\tag{2}$$

where i runs over all massive particles which are involved in the transition $A \to B$. Here, it is understood that the differential operator does not act on the m_i appearing in coupling constants, since this would generate tree-level interactions involving the Higgs boson that do not exist in the SM. Special care must be exercised if this low-energy theorem is to be applied beyond the leading order. Then it must be formulated for the bare quantities of the theory. The renormalization is performed after the left-hand side of Eq. (2) has been evaluated.

This paper is organized as follows. In Sect. 2, we derive a heavy-top-quark effective Lagrangian for the ZZH interaction, including the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections, by means of the low-energy theorem. From this Lagrangian we may instantly read off the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to $\Gamma(H \to ZZ)$, which, to our knowledge, is a new result. In order to obtain the corresponding corrections to $\Gamma\left(Z \to f\bar{f}H\right)$ and $\sigma(e^+e^- \to ZH)$, we also need to include similar corrections originating in the gauge sector. This will be done in Sect. 3 by invoking the so-called improved Born approximation (IBA) [16]. Section 4 contains our conclusions.

2 Effective Lagrangian

We use dimensional regularization in $n = 4 - 2\epsilon$ space-time dimensions and introduce a 't Hooft mass, μ, to keep the coupling constants dimensionless. As usual, we take γ_5 to be anticommuting. We work in the on-shell renormalization scheme [17], with G_F as a basic parameter.

Prior to actually evaluating loop amplitudes, we develop the general formalism. The starting point of our analysis is the amplitude characterizing the propagation of an on-shell Z boson in the presence of quantum effects due to a virtual high-mass top quark,

$$\mathcal{M}(Z \to Z) = (M_Z^0)^2 - \Pi_{ZZ}(q^2)\big|_{q^2=(M_Z^0)^2} , \tag{3}$$

where $\Pi_{ZZ}(q^2)$ is the unrenormalized transverse self-energy of the Z boson, with momentum q, and is expressed in terms of bare parameters. Here and in the following, bare parameters are marked by the superscript 0. In the G_F representation, $\Pi_{ZZ}(q^2)$ is proportional to $(M_Z^0)^2$, which originates from the two $t\bar{t}Z$ gauge couplings. Apart from this prefactor, we may put $q^2 = 0$ in Eq. (3), since we are working in the high-m_t approximation. The low-energy theorem (2) now tells us that we may attach a zero-momentum Higgs boson to the $Z \to Z$ transition amplitude by carrying out the operation

$$\lim_{p_H \to 0} \mathcal{M}(Z \to Z + H) = \frac{1}{v^0}\left(\frac{m_t^0 \partial}{\partial m_t^0} + \frac{M_Z^0 \partial}{\partial M_Z^0}\right)\mathcal{M}(Z \to Z), \tag{4}$$

where we must treat the overall factor $(M_Z^0)^2$ of $\Pi_{ZZ}(0)$ in Eq. (3) as a constant. This leads us to

$$\lim_{p_H \to 0} \mathcal{M}(Z \to Z + H) = \frac{2(M_Z^0)^2}{v^0}(1 + E), \tag{5}$$

3

with

$$E = -\frac{(m_t^0)^2 \partial}{\partial (m_t^0)^2} \frac{\Pi_{ZZ}(0)}{(M_Z^0)^2}. \tag{6}$$

We are now in the position to write down the heavy-top-quark effective ZZH interaction Lagrangian,

$$\mathcal{L}_{ZZH} = (M_Z^0)^2 (Z^0)_\mu (Z^0)^\mu \frac{H^0}{v^0} (1 + E). \tag{7}$$

Then, we have to carry out the renormalization procedure, i.e., we have to split the bare parameters into renormalized ones and counterterms. We fix the counterterms according to the on-shell scheme. In the case of the Z-boson mass and wave function, we have

$$\begin{aligned} (M_Z^0)^2 &= M_Z^2 + \delta M_Z^2, \\ (Z^0)^\mu &= (1 + \delta Z_Z)^{1/2} Z^\mu, \end{aligned} \tag{8}$$

with

$$\begin{aligned} \delta M_Z^2 &= \Pi_{ZZ}(0), \\ \delta Z_Z &= -\Pi'_{ZZ}(0), \end{aligned} \tag{9}$$

where we have neglected M_Z against m_t in the loop amplitudes. For dimensional reasons, δZ_Z does not receive corrections in $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$. From the analysis of $\Gamma\left(H \to f\bar{f}\right)$ in $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ [18] we know that

$$\frac{H^0}{v^0} = 2^{1/4} G_F^{1/2} H (1 + \delta_u), \tag{10}$$

with

$$\delta_u = N_c x_t \left[\frac{7}{6} - C_F \frac{\alpha_s}{\pi} \left(\frac{\zeta(2)}{2} + \frac{3}{4} \right) \right], \tag{11}$$

where $C_F = (N_c^2 - 1)/(2N_c) = 4/3$. δ_u is a universal correction, which occurs as a building block in the renormalizations of all Higgs-boson production and decay processes. Putting everything together, we obtain the renormalized version of Eq. (7),

$$\mathcal{L}_{ZZH} = 2^{1/4} G_F^{1/2} M_Z^2 Z_\mu Z^\mu H (1 + \delta_{ZZH}), \tag{12}$$

with

$$\delta_{ZZH} = \delta_u + \frac{\delta M_Z^2}{M_Z^2} + E. \tag{13}$$

In order for δ_{ZZH} to be finite through $\mathcal{O}(\alpha_s G_F m_t^2)$, we still need to renormalize the top-quark mass in the $\mathcal{O}(G_F m_t^2)$ expressions for $\delta M_Z^2/M_Z^2$ and E, i.e., we need to substitute

$$m_t^0 = m_t + \delta m_t, \tag{14}$$

with [18, 19]

$$\frac{\delta m_t}{m_t} = -\frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu^2}{m_t^2} \right)^\epsilon \Gamma(1+\epsilon) \frac{3-2\epsilon}{\epsilon(1-2\epsilon)}, \tag{15}$$

where Γ is Euler's gamma function.

Now, we turn to the evaluation of the two-loop amplitudes. To simplify the notation, we introduce $t = m_t^2$ and $Z = \delta M_Z^2 / M_Z^2$. We label $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ contributions with the subscripts 1 and 2, respectively. Quantities with (without) the superscript 0 are written in terms of m_t^0 (m_t). First, we shall list the $\mathcal{O}(G_F m_t^2)$ results. These may be extracted from Ref. [20] and read

$$Z_1 = N_c x_t \left(\frac{4\pi \mu^2}{m_t^2} \right)^\epsilon \Gamma(1 + \epsilon) \left(-\frac{2}{\epsilon} + \mathcal{O}(\epsilon) \right), \tag{16}$$

$$E_1 = N_c x_t \left(\frac{4\pi \mu^2}{m_t^2} \right)^\epsilon \Gamma(1 + \epsilon) \left(\frac{2}{\epsilon} - 2 + \mathcal{O}(\epsilon) \right). \tag{17}$$

In Ref. [20], E_1 has been computed diagrammatically. This allows us to check Eq. (6) in $\mathcal{O}(G_F m_t^2)$. In fact, one immediately verifies that

$$E_1 = -\frac{t\partial}{\partial t} Z_1. \tag{18}$$

The QCD corrections to the electroweak-gauge-boson vacuum polarizations have been evaluated by means of dispersion relations in Ref. [21]. These results may be converted to dimensional regularization by adjusting the ultraviolet regulators as described in Refs. [18, 22]. In the case of Z_2, this leads to

$$Z_2 = N_c C_F \frac{\alpha_s}{\pi} x_t \left(\frac{4\pi \mu^2}{m_t^2} \right)^{2\epsilon} \Gamma^2(1 + \epsilon) \left(\frac{3}{2\epsilon^2} + \frac{11}{4\epsilon} + \frac{31}{8} + \mathcal{O}(\epsilon) \right), \tag{19}$$

which agrees with the result obtained in last two papers of Ref. [23]. Notice that Eq. (19) already contains the contribution proportional to δm_t which emerges from the renormalization of the top-quark mass in Eq. (16). Our final aim is to compute E_2. According to Eq. (6), we have

$$E_2^0 = -\frac{t\partial}{\partial t} Z_2^0. \tag{20}$$

Furthermore, we have

$$Z_2 = Z_2^0 + \delta Z_2, \tag{21}$$

$$E_2 = E_2^0 + \delta E_2, \tag{22}$$

where the counterterms are obtained by scaling the one-loop results,

$$\delta Z_2 = \frac{\delta t}{t} \frac{t\partial}{\partial t} Z_1, \tag{23}$$

$$\delta E_2 = \frac{\delta t}{t} \frac{t\partial}{\partial t} E_1. \tag{24}$$

Substituting Eq. (21) in Eq. (20) and using Eq. (23), we obtain

$$E_2^0 = -\frac{t\partial}{\partial t} Z_2 + \frac{t\partial}{\partial t} \left(\frac{\delta t}{t} \frac{t\partial}{\partial t} Z_1 \right). \tag{25}$$

On the other hand, inserting Eq. (18) into Eq. (24) yields

$$\delta E_2 = -\frac{\delta t}{t}\left(\frac{t\partial}{\partial t}\right)^2 Z_1. \tag{26}$$

Now, combining Eqs. (25) and (26), we obtain

$$E_2 = -\frac{t\partial}{\partial t}Z_2 + \left(\frac{t\partial}{\partial t}\frac{\delta t}{t}\right)\frac{t\partial}{\partial t}Z_1. \tag{27}$$

Using Eq. (18) together with

$$\frac{t\partial}{\partial t}\frac{\delta t}{t} = -\epsilon\frac{\delta t}{t}, \tag{28}$$

which may be gleaned from Eq. (15), this becomes

$$E_2 = -\frac{t\partial}{\partial t}Z_2 + \epsilon\frac{\delta t}{t}E_1. \tag{29}$$

Obviously, knowledge of the $\mathcal{O}(\epsilon)$ term of E_1 is not necessary for our purposes. Using Eqs. (15), (17), and (19), we obtain from Eq. (29) the desired two-loop three-point amplitude,

$$E_2 = N_c C_F \frac{\alpha_s}{\pi} x_t \left(\frac{4\pi\mu^2}{m_t^2}\right)^{2\epsilon} \Gamma^2(1+\epsilon)\left(-\frac{3}{2\epsilon^2} - \frac{11}{4\epsilon} + \frac{5}{8} + \mathcal{O}(\epsilon)\right). \tag{30}$$

Finally, adding up Eqs. (11), (16), (17), (19), and (30), we find the ultraviolet-finite correction in Eq. (12),

$$\delta_{ZZH} = N_c x_t \left[-\frac{5}{6} + C_F \frac{\alpha_s}{\pi}\left(-\frac{\zeta(2)}{2} + \frac{15}{4}\right)\right]$$
$$\approx -\frac{5}{2}x_t\left(1 - 4.684\frac{\alpha_s}{\pi}\right). \tag{31}$$

This completes the derivation of the effective ZZH interaction Lagrangian. We recover the notion that, in the electroweak on-shell renormalization scheme implemented with G_F, the $\mathcal{O}(G_F m_t^2)$ terms generally are screened by their QCD corrections. We are not aware of any counterexample to this rule in electroweak physics. In the present case, the reduction in size amounts to approximately -16% for $\alpha_s = 0.108$, which is the value of $\alpha_s(\mu)$ at $\mu = m_t = 174$ GeV if $\alpha_s(M_Z) = 0.118$ [24].

As a corollary, we note that, in the G_F formulation of the on-shell scheme, the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to $\Gamma(H \rightarrow ZZ)$ appear as the overall factor $(1 + \delta_{ZZH})^2$. This reproduces the well-known one-loop result [20]. Of course, in order for the high-m_t approximation to be valid in this case, $2M_Z < M_H \ll m_t$ must be satisfied, which is probably not very realistic.

3 Higgs production at LEP

Armed with the high-m_t effective ZZH interaction Lagrangian derived in the previous section, we now proceed to the analysis of the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to $\Gamma\left(Z \to f\bar{f}H\right)$ and $\sigma(e^+e^- \to ZH)$. Detailed inspection of the one-loop results [7, 9] reveals that, apart from the ZZH vertex, leading high-m_t corrections also originate in the renormalizations of the Z-boson propagator and the $Z\gamma$ mixing amplitude. The loop-induced $Z\gamma H$ coupling does not produce such a correction, since it does not involve $t\bar{t}Z$ axial couplings. Furthermore, the top quark does not yet enter the residual vertex and box corrections at one loop, so that $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections do not arise here either.

The so-called improved Born approximation (IBA) [16] provides a systematic and convenient method to incorporate the leading high-m_t corrections to processes within the gauge sector of the SM. The recipe is as follows. Starting from the Born formula expressed in terms of c_w, s_w, and the fine-structure constant defined in Thomson scattering, α, one substitutes

$$\alpha \to \overline{\alpha} = \frac{\alpha}{1-\Delta\alpha}, \qquad c_w^2 \to \overline{c}_w^2 = 1 - \overline{s}_w^2 = c_w^2(1-\Delta\rho), \tag{32}$$

where $\overline{\alpha}$ is the effective fine-structure constant measured at the Z-boson scale and $\Delta\rho = N_c x_t$ is the shift in the ρ parameter induced by the top quark. To eliminate $\overline{\alpha}$ in favour of G_F, one exploits the relation

$$\frac{\overline{\alpha}}{\overline{c}_w^2 \overline{s}_w^2} = \frac{\sqrt{2}}{\pi} G_F M_Z^2 \frac{1}{1-\Delta\rho}, \tag{33}$$

which correctly accounts for the dominant m_t power terms as well as the leading logarithms that trigger the running of the fine-structure constant.

In Ref. [7], it has been explained how the IBA may be combined with specific knowledge of the high-m_t behaviour of the ZZH vertex [20] to find the $\mathcal{O}(G_F m_t^2)$ correction to $\Gamma\left(Z \to f\bar{f}H\right)$. Specifically, the correction factor relative to the Born formula of $\Gamma\left(Z \to f\bar{f}H\right)$ written with G_F, as in Eqs. (2) and (3) of Ref. [7],[1] is given by

$$\begin{aligned}
1 + \Delta_{f\bar{f}ZH} &= (1+\delta_{ZZH})^2 \frac{\overline{\alpha}/\left(\overline{c}_w^2 \overline{s}_w^2\right)}{\sqrt{2}G_F M_Z^2/\pi} \frac{\overline{v}_f^2 + a_f^2}{v_f^2 + a_f^2} \\
&= 1 + 2\delta_{ZZH} + \left(1 - \frac{8c_w^2 Q_f v_f}{v_f^2 + a_f^2}\right)\Delta\rho,
\end{aligned} \tag{34}$$

where $\overline{v}_f = 2I_f - 4\overline{s}_w^2 Q_f$, and we have omitted terms of $\mathcal{O}(G_F^2 m_t^4)$ in the second line. In fact, this reproduces Eq. (1). Equation (34), with $f = e$, is also the correct $\mathcal{O}(G_F m_t^2)$ correction factor for $\sigma(e^+e^- \to ZH)$ [6, 9], provided that the Born result is written with G_F, as in Eq. (4.7) of Ref. [9].

---

[1] We use this opportunity to correct a misprint in the published version of Ref. [7], which is absent in the preprint. The factor $\sqrt{x/4 - x}$ in the last term of Eq. (3) should be replaced by $\sqrt{x/(4-x)}$.

This procedure readily carries over to $\mathcal{O}(\alpha_s G_F m_t^2)$. We just need to include in Eq. (34) the corresponding terms of δ_{ZZH} and $\Delta\rho$. In the case of $\Delta\rho$, one has [21, 23]

$$\Delta\rho = N_c x_t \left[1 - C_F \frac{\alpha_s}{\pi} \left(\zeta(2) + \frac{1}{2} \right) \right]. \tag{35}$$

Inserting Eqs. (31) and (35) into Eq. (34), we obtain our final result,

$$\Delta_{f\bar{f}ZH} = N_c x_t \left\{ -\frac{2}{3} \left[1 + C_F \frac{\alpha_s}{\pi} \left(3\zeta(2) - \frac{21}{2} \right) \right] - \frac{8c_w^2 Q_f v_f}{v_f^2 + a_f^2} \left[1 - C_F \frac{\alpha_s}{\pi} \left(\zeta(2) + \frac{1}{2} \right) \right] \right\}$$
$$\approx -2x_t \left[1 - 7.420 \frac{\alpha_s}{\pi} + \frac{12c_w^2 |Q_f|(1 - 4s_w^2 |Q_f|)}{(1 - 4s_w^2 |Q_f|)^2 + 1} \left(1 - 2.860 \frac{\alpha_s}{\pi} \right) \right]. \tag{36}$$

Using $M_W = 80.24\,\text{GeV}$, $M_Z = 91.19\,\text{GeV}$, and $\alpha_s = 0.108$, we find that strong-interaction effects modify the magnitude of $\Delta_{f\bar{f}ZH}$ in Eq. (1) by roughly -26%, -18%, -15%, and -16% for neutrinos, charged leptons, up-type quarks, and down-type quarks, respectively, i.e., we observe an appreciable screening in all cases. In comparison, we remark that the relative QCD correction in Eq. (35) is merely -10% for the same α_s value.

The examples considered here give support to the heuristic rule that, in the electroweak on-shell scheme formulated with G_F, the $\mathcal{O}(G_F m_t^2)$ terms are screened by their QCD corrections. By the same token, this screening is weakened—or possibly converted into antiscreening—when the top-quark mass is renormalized according to the $\overline{\text{MS}}$ scheme, with the choice $\mu = \mathcal{O}(m_t)$. This may be understood by observing that, for $\mu = m_t$, the $\overline{\text{MS}}$ mass [19],

$$\overline{m}_t(\mu) = m_t \left[1 + C_F \frac{\alpha_s}{\pi} \left(\frac{3}{4} \ln \frac{m_t^2}{\mu^2} - 1 \right) \right], \tag{37}$$

is smaller than m_t, the reduction of x_t being approximately 9% for $\alpha_s = 0.108$. Equation (37) follows on from Eq. (15) by discarding the poles in ϵ. Consequently, the negative QCD corrections are partly absorbed into x_t, as we pass from the on-shell scheme to the $\overline{\text{MS}}$ scheme. In fact, the relative QCD corrections are then only -16% for $\Gamma(Z \to \nu\bar{\nu}H)$, -9% for $\Gamma(Z \to \ell^+\ell^-H)$ and $\sigma(e^+e^- \to ZH)$, -7% for $\Gamma\left(Z \to d\bar{d}H\right)$ and $\Gamma(H \to ZZ)$, and -6% for $\Gamma(Z \to u\bar{u}H)$, where ν, ℓ, u, and d are generic neutrinos, charged leptons, up-type quarks, and down-type quarks, respectively.

4 Conclusions

The quantum corrections to the production and decay processes of the SM Higgs boson are now well established in the one-loop approximation [5]. Since experiments seem to favour a high-mass top quark [12], one is led to focus attention on the leading high-m_t terms, which are of $\mathcal{O}(G_F m_t^2)$, and one would like to gain control over the dominant shifts in these terms due to higher-order effects, which are of $\mathcal{O}(G_F^2 m_t^4)$ and $\mathcal{O}(\alpha_s G_F m_t^2)$. Some work in that direction has already been done. The $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections have been evaluated for the Higgs-boson decays into fermions [18] and, in particular, into bottom

quarks [15, 25]. Furthermore, the leading top-quark-induced $\mathcal{O}(\alpha_s^2)$ corrections to the Higgs-boson decays into quarks have been found [26]. In this context, we should also mention the $\mathcal{O}(\alpha_s)$ [14, 27] and $\mathcal{O}(G_F m_t^2)$ [28] corrections to the Higgs-boson decay into gluons, which is mediated by a top-quark loop.

In this article, we continued this research program by presenting the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to $\Gamma(H \rightarrow ZZ)$, $\Gamma\left(Z \rightarrow f\bar{f}H\right)$, and $\sigma(e^+e^- \rightarrow ZH)$. As in a previous work [15], we took advantage of a low-energy theorem, which allowed us to reduce the task of solving two-loop three-point integrals to a two-loop two-point problem. Our results are in line with all previous studies of $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to electroweak processes, which have always shown that such corrections screen the leading m_t dependence of the one-loop results. For $\alpha_s = 0.108$, the screening effects amount to roughly -26% for $\Gamma\left(Z \rightarrow \nu\bar{\nu}H\right)$, -18% for $\Gamma\left(Z \rightarrow \ell^+\ell^-H\right)$ and $\sigma(e^+e^- \rightarrow ZH)$, -16% for $\Gamma\left(Z \rightarrow d\bar{d}H\right)$ and $\Gamma(H \rightarrow ZZ)$, and -15% for $\Gamma\left(Z \rightarrow u\bar{u}H\right)$, where ν, ℓ, u, and d are generic neutrinos, charged leptons, up-type quarks, and down-type quarks, respectively.

References

[1] J.D. Bjorken, in *Weak Interactions at High Energy and the Production of New Particles: Proceedings of Summer Institute on Particle Physics*, August 2–13, 1976, ed. by M.C. Zipf, SLAC Report No. 198 (1976) p. 1.

[2] P. Janot, LAL Report No. 94–59 (August 1994).

[3] J. Ellis, M.K. Gaillard, and D.V. Nanopoulos, Nucl. Phys. **B106** (1976) 292.

[4] B.L. Ioffe and V.A. Khoze, Fiz. Elem. Chastits At. Yadra **9** (1978) 118 [Sov. J. Part. Nucl. **9** (1978) 50].

[5] B.A. Kniehl, Phys. Rep. **240** (1994) 211.

[6] J. Fleischer and F. Jegerlehner, Nucl. Phys. **B228** (1983) 1;
S. Dawson and H.E. Haber, Phys. Rev. **D44** (1991) 53.

[7] B.A. Kniehl, DESY Report No. 92–021 (February 1992); Phys. Lett. **B282** (1992) 249.

[8] R. Kleiss, Phys. Lett. **141B** (1984) 261;
F.A. Berends and R. Kleiss, Nucl. Phys. **B260** (1985) 32.

[9] B.A. Kniehl, Z. Phys. **C55** (1992) 605.

[10] J. Fleischer and F. Jegerlehner, Nucl. Phys. **B216** (1983) 469;
A. Denner, J. Küblbeck, R. Mertig, and M. Böhm, Z. Phys. **C56** (1992) 261.

[11] E. Gross, B.A. Kniehl, and G. Wolf, Z. Phys. **C63** (1994) 417; erratum (to appear).

[12] F. Abe et al. (CDF Collaboration), Phys. Rev. **D50** (1994) 2966; Phys. Rev. Lett. **73** (1994) 225.

[13] A.I. Vaĭnshteĭn, M.B. Voloshin, V.I. Zakharov, and M.A. Shifman, Yad. Fiz. **30** (1979) 1368 [Sov. J. Nucl. Phys. **30** (1979) 711];
A.I. Vaĭnshteĭn, V.I. Zakharov, and M.A. Shifman, Usp. Fiz. Nauk **131** (1980) 537 [Sov. Phys. Usp. **23** (1980) 429];
L.B. Okun, *Leptons and Quarks*, (North-Holland, Amsterdam, 1982) p. 229;
M.B. Voloshin, Yad. Fiz. **44** (1986) 738 [Sov. J. Nucl. Phys. **44** (1986) 478];
M.A. Shifman, Usp. Fiz. Nauk **157** (1989) 561 [Sov. Phys. Usp. **32** (1989) 289];
J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunter's Guide*, (Addison-Wesley, Redwood City, 1990) p. 40.

[14] A. Djouadi, M. Spira, and P.M. Zerwas, Phys. Lett. **B264** (1991) 440;
S. Dawson, Nucl. Phys. **B359** (1991) 283.

[15] B.A. Kniehl and M. Spira, Nucl. Phys. **B432** (1994) 39.

[16] M. Consoli, W. Hollik, and F. Jegerlehner, in *Z Physics at LEP 1*, ed. by G. Altarelli, R. Kleiss, and C. Verzegnassi, CERN Yellow Report No. 89–08 (1989) Vol. 1, p. 7;
W.F.L. Hollik, Fortschr. Phys. **38** (1990) 165;
F. Halzen, B.A. Kniehl, and M.L. Stong, in *Particle Physics: VI Jorge André Swieca Summer School*, Campos de Jordão, Brasil, 14–26 January, 1991, ed. by O.J.P. Éboli, M. Gomes, and A. Santoro (World Scientific, Singapore, 1992) p. 219; Z. Phys. **C58** (1993) 119.

[17] K-I. Aoki, Z. Hioki, R. Kawabe, M. Konuma, and T. Muta, Prog. Theor. Phys. **64** (1980) 707; Prog. Theor. Phys. **65** (1981) 1001; Prog. Theor. Phys. Suppl. **73** (1982) 1;
M. Böhm, H. Spiesberger, and W. Hollik, Fortschr. Phys. **34** (1986) 687;
A. Denner, Fortschr. Phys. **41** (1993) 307.

[18] B.A. Kniehl and A. Sirlin, Phys. Lett. **B318** (1993) 367;
B.A. Kniehl, Phys. Rev. **D50** (1994) 3314.

[19] R. Tarrach, Nucl. Phys. **B183** (1981) 384.

[20] Z. Hioki, Phys. Lett. **B224** (1989) 417;
B.A. Kniehl, Nucl. Phys. **B352** (1991) 1.

[21] B.A. Kniehl, Nucl. Phys. **B347** (1990) 86.

[22] B.A. Kniehl and A. Sirlin, Phys. Rev. **D47** (1993) 883.

[23] A. Djouadi and C. Verzegnassi, Phys. Lett. **B195** (1987) 265;
A. Djouadi, Nuovo Cim. **100A** (1988) 357;

A. Djouadi and P. Gambino, Phys. Rev. **D49** (1994) 3499; Phys. Rev. **D49** (1994) 4705.

[24] S. Bethke, to appear in *Proceedings of the Tennessee International Symposium on Radiative Corrections: Status and Outlook*, Gatlinburg, Tennessee, June 27–July 1, 1994, ed. by B.F.L. Ward (World Scientific, Singapore, 1995).

[25] A. Kwiatkowski and M. Steinhauser, Phys. Lett. **B338** (1994) 66; erratum (to appear).

[26] K.G. Chetyrkin, J.H. Kühn, and A. Kwiatkowski, Karlsruhe University Report No. TTP94–10, hep–ph/9407271 (July 1994), to appear in *Proceedings of the XXIXth Rencontres de Moriond: QCD and High Energy Interactions*, Méribel, France, March 19–26, 1994;
B.A. Kniehl, MPI Report No. MPI/PhT/94–69 (October 1994), Phys. Lett. B (to appear).

[27] T. Inami, T. Kubota, and Y. Okada, Z. Phys. **C18** (1983) 69.

[28] A. Djouadi and P. Gambino, Phys. Rev. Lett. **73** (1994) 2528.